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OCTIC B-SPLINE COLLOCATION SOLUTION WITH NON-UNIFORM LENGTH FOR EIGHTH ORDER LINEAR DIFFERENTIAL EQUATION

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Abstract

Presentation of Numerical solution for eighth order linear boundary value problem using Octic B-spline collocation method with non-uniform length is the subject of this paper. In this approach recursive form of B-spline function is used as basis in collocation method. Numerical examples are considered to show the advantage of recursive of B-spline function particularly in non-fixing the length of subintervals.

Keywords: B-Spline; Collocation; Recursive; Linear Differential Equation; Octic.

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1. Introduction

The general eighth order linear differential equation with the boundary conditions is given as

$$P0(x)\frac{d^8U}{dx^8}+P_1(x)\frac{d^7U}{dx^7}+P2(x)\frac{d^6U}{dx^6}+P3(x)\frac{d^5U}{dx^5}+P_4(x)\frac{d^4U}{dx^4}+P5(x)\frac{d^3U}{dx^3}+P_6(x)\frac{d^2U}{dx^2}+P_7(x)\frac{dU}{dx}+P_8(x)U=Q(x)$$
$$x\in(a,b), \qquad (1)$$

with the boundary conditions

$$U(a)=d1, U(b)=d2\quad U'(a)=d3, U'(b)=d4, U''(a)=d5, U''(b)=d6, U'''(a)=d7, U'''(b)=d8 \qquad (2)$$

where $a,b,d_1,d_2,d_3,d_4,d_5,d_6,d_7,d8$ **are constants.** $P_0(x),P_1(x),P_2(x)$, $P_3(x),P_4(x)$, $P_5(x),P_6(x)$, $P_7(x),P_8(x),Q(x)$ **are function of** x.

Solving such higher order linear differential equations and getting exact solutions is sometimes difficult. Authors developed methods to obtain numerical methods. Some of the selected numerical methods are mentioned. Finite difference method is used to solve such equations by Boutayeb and Twizell [1] .Vishwanadam and Ballem [2] used Galerkin method with quintic B-spline, Siddiqi and iftikhar [3] solved by using homotopy analysis method.

In this paper, A Octic degree B-spline based collocation method has been elaborated for the solution of linear eighth order differential equation with boundary conditions defined in Eq.(1) with Eq.(2). for non-fixed length.

2. The Numerical Scheme

Let $[a,b]$ be the domain of the governing differential equation and is partitioned as $X = \{a = x_0, x_1, x_2 ....x_{n-1}, x_n = b\}$ without any restriction on length of n sub domains. Let $N_i(x)$ be Octic B- splines with the knots at the points x_i, i=0,1,…n. The set { $N_{-8}, N_{-7}, N_{-6}, \ldots, N_6, N_7, N8$ } forms a basis for functions defined over [a,b].

Let $$U^h(x) = \sum_{i=-8}^{n+8} C_i N_{i,p}(x) \tag{3}$$

where C_i's are constants to be determined and $N_{i,p}(x)$ are the Octic B-spline functions , be the approximate global solution to the exact solution $U(x)$ of the considered eighth order linear differential equation (1). A zero degree and other than zero degree B-spline basis functions [5, 6] are defined at x_i recursively over the knot vector space $X = \{x_1, x_2, x_3 ....x_{n-1}, x_n\}$ as

$i)\quad if \quad p = 0$

$N_{i,p}(x) = 1 \qquad if \quad x \in (x_i, x_{i+i})$

$N_{i,p}(x) = 0 \qquad if \quad x \notin (x_i, x_{i+i})$

$ii)\quad if \quad p \geq 1$

$$N_{i,p}(x) = \frac{x - x_i}{x_{i+p} - x_i} N_{i,p-1}(x) + \frac{x_{i+p+1} - x}{x_{i+p+1} - x_{i+1}} N_{i+1,p-1}(x) \tag{4}$$

where p is the degree of the B-spline basis function and x is the parameter belongs to X .When evaluating these functions, ratios of the form 0/0 are defined as zero.

Derivatives of B-splines

If p=8, we have

$$N'_{i,p}(x) = \frac{x - x_i}{x_{i+p} - x_i} N'_{i,p-1}(x) + \frac{N_{i,p-1}(x)}{x_{i+p} - x_i} + \frac{x_{i+p+1} - x}{x_{i+p+1} - x_{i+1}} N'_{i+1,p-1}(x) - \frac{N_{i+1,p-1}(x)}{x_{i+p+1} - x_{i+1}}$$

$$N^{viii}{}_{i,p}(x) = 8\frac{N^{vii}{}_{i,p-1}(x)}{x_{i+p} - x_i} - 8\frac{N^{vii}{}_{i+1,p-1}(x)}{x_{i+p+1} - x_{i+1}} \tag{5}$$

$$(U^h)^{viii}(x) = \sum_{i=-8}^{n+8} C_i N^{viii}{}_{i,p}(x) \tag{6}$$

The $x_i's$ are known as nodes, the nodes are treated as knots in collocation B-spline method where the B-spline basis functions are defined and these nodes are used to make the residue equal to zero to determine unknowns $C_i's$ in (2).Eight extra knots are taken into consideration besides the domain of problem to maintain the partition of unity[7] when evaluating the Octic B-spline basis functions at the nodes which are within the considered domain.

Substituting the equations (3) and (6) in equation (1) for U (x) and derivatives of U (x).Then system of (n+1) linear equations are obtained in (n+8) constants. Applying the boundary conditions to equation (2), eight more equations are generated in constants. Finally, we have (n+9) equations in (n+9) constants.

Solving the system of equations for constants and substituting these constants in equation (3) then assumed solution becomes the known approximation solution for equation (1) at corresponding the collocation points. This is implemented using the Matlab programming.

Numerical Experiment

A linear eighth order differential equation with boundary conditions is considered for testing the accuracy of the proposed method.
Example

$$\frac{d^8y}{dx^8} + \frac{d^7y}{dx^7} + 2\frac{d^6y}{dx^6} + 2\frac{d^5y}{dx^5} + \frac{d^4y}{dx^4} + 2x\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + x^2\frac{dy}{dx} + xy = -(-27 + 14x - 2x^3 + x^4)\cos x - (-3x^3 - 13x^2 + 11x + 17)\sin x \qquad 0 < x < 1$$

with the boundary conditions $y(0) = 0$ $y(1) = 0$ $y'(0) = 1$ $y'(1) = -e$ $y''(0) = 0$ $y''(1) = -4e$ $y'''(0) = -3$ $y'''(1) = -9e$

The exact solution for the given problem is $y = (x^2 - 1)\sin x$

Table1: Comparison of Octic B-spline collocation solution with Exact solution

x	0	.100	.25	.38	.45	.6	.79	.82	.91	1
OBCS*	0.000	-0.0988	-0.2319	-0.3173	-0.3468	-0.3613	-0.2670	-0.2395	-0.1357	0.0000
Exact solution	0.000	-0.0988	-0.2319	-0.3174	-0.3469	-0.3614	-0.2670	-0.2395	-0.1357	0.0000

OBCS* Octic B-spline collocation solution

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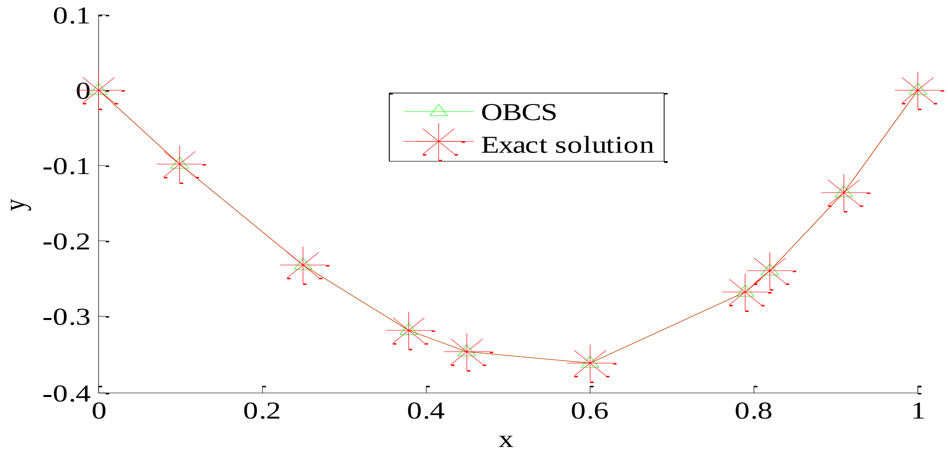

Figure1 Comparison of Octic B-spline Collocation solution with the exact solution

Collocation method using the Octic degree B-splines as basis functions are applied for the present problem for 10 collocation points. Octic B-spline collocation solution and exact solution are presented in table1.Grapical representation of both the solutions are shown in figure1.Numerical solution values are almost equal to the exact values.

Absolute relative errors at the collocation points are shown graphically in figure2

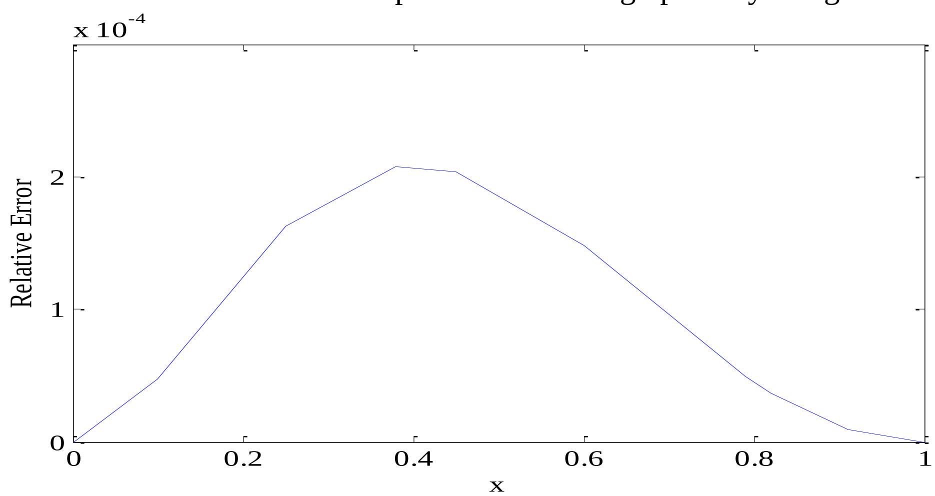

Figure2 Absolute Relative errors for example problem

3. Conclusion

In this paper, collocation method using the B-splines as basis function has been developed to approximate the linear eighth-order boundary value problems with variable coefficients. In this method, the assumed approximate solution is directly substituted in given differential equation and evaluated unknown values in that approximate solution with the help of collocation points and boundary conditions. It is observed that the approximate values is almost equal to exact values and consequently approximate values converging to the exact values by increasing the number of collocation points. Also implementing of proposed method is very easy.

References

[1] Boutayeb,Twizell, E.H : Finite difference methods for the solution of eighth order boundary value problems.Int.J.Comput.Math.48,63-75 (1993)
[2] Vishwanadam, Ballem : Numerical solution of eighth order boundary value problems by Galarekin method with quintic B-splines.Int.J.Comput.Appl.89(15),7-13(2014)
[3] Siddiqi, Iftikhar,M :Numerical solution of higher order boundary value problems.Abstr.Appl.Ann.(2013)
[4] Zaffer Elahi, Ghazala Akram, Shahid Saeed Siddiqi: Numerical solution for solving special eighth order higher order linear boundary value problems using Legender Galerkin Method.J.Math sci (2016),Springer link.com
[5] I. J. SCHOENBERG Contributions to the problem of approximation of equidistant data by analytic functions, Quart. Appl. Math. 4 (1946), 45-99; 112-141.
[6] H. B. CURRYA ND I. J. SCHOENBERG On Polya frequency functions IV: The fundamental spline functions and their limits, J. Anal. Math. 17 (1966), 71-107.
[7] CARL DE BOOR On Calculating with B-plines. JOURNAL OF APPROXIMATION THEORY, SO-62 (1972).

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